

The Cone Geometry of ObjID 126 (Delos): Latitude from the Engraved Network

A Geometrically Explicit Estimation with Stated Assumptions and Defendable Confidence Intervals

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ABSTRACT

We analyse the high-resolution 3D photogrammetric surface model of the exceptionally well-preserved ancient Hellenistic conic sundial ObjID 126 from the Archaeological Museum of Delos. The artifact preserves eleven engraved hour lines, all three seasonal day-lines (equinox and both solstices), and 31 verified curve-to-curve intersections with day-circle fit residuals of 0.35–0.42 mm RMS. We demonstrate that the engraved hour network carries a uniform $+5.27\% \pm 0.34\%$ angular-unit scale distortion, which is isolated and calibrated against the theoretical equinoctial design invariant $H_E = 90^\circ$. By pairing each solstitial day-circle’s placement with its calibrated half-day hour arc, we recover the intended latitude independently of epoch obliquity. A joint measurement-level bootstrap and hierarchical profile-likelihood estimation over four independent construction witnesses (cone axis, upper front face, summer division, and winter division) yields a defended design latitude of $\hat{\varphi} = 38.01^\circ$ (68% CI $[37.32^\circ, 38.68^\circ]$, artisan spread $\tau = 1.26^\circ$), corresponding to $s = 46; 54$ for $g = 60$, matching the Delos (37.40°) and Athens (37.97°) climate band.

KEYWORDS conic sundials · 3D photogrammetry · Delos · ObjID 126 · angular unit calibration · epsilon-free latitude recovery · profile likelihood

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i Supplement S6

Supplement S6 to the Ancient Sundials repository (Berlin Sundial Collaboration, ancient-sundials.lettire.ai). This draft documents a worked example for the object page of ObjID 126 (Delos). It is a preprint draft and has not been peer reviewed. Where a figure in the repository differs from a figure here, the repository, which is rebuilt from the current database, is authoritative.

The register’s consensus latitude for ObjID 126 is 37.97 degrees and this paper gives 38.01 degrees; the two values have not been reconciled. Their difference of 0.04 degrees lies well inside the 68 % interval stated here.

1 Introduction: The Exceptional Delos Conic Dial

Among the corpus of surviving ancient Greek sundials, Dialface ObjID 126, preserved in the Archaeological Museum of Delos, represents a masterwork of Hellenistic stonecutting and mathematical design [1], [2].

The artifact preserves all primary features intact: eleven hour lines, three seasonal day-line circles, dressed upper and lower front planes, and 31 curve-to-curve intersections. The photogrammetric mesh fits the day-circles to within 0.35–0.42 mm RMS across up to 322 groove centreline points per curve.

CONCEPT The research objective for ObjID 126

Because measurement noise on ObjID 126 is negligible (< 0.4 mm), the investigation is not limited by scan resolution. The scientific question is purely how much design information the engraved stone carries, and how to deduce its intended construction proportions without distortion.

2 Functional Relations of the Nodus-on-Axis Model

Following the analytical framework of the keystone geometry paper [3], the instrument's features obey four exact relations:

1. **(R1) Day-Circle Axial Placement:** A noon shadow ray from the nodus at declination δ strikes the cone at axial distance:

$$z(\delta) = \frac{d}{1 - \tan \alpha \tan \delta}. \quad (1)$$

Reading. The axial distance $z(\delta)$ from the apex equals the scale d divided by $1 - \tan \alpha \tan \delta$, giving $\ell_E = d$, $\ell_S = d/(1 - x)$, and $\ell_W = d/(1 + x)$ with $x = \tan \alpha \tan \varepsilon$.

2. **(R2) Cone Cavity Geometry:** Each day-line circle has radius $r = z \tan \alpha$. Direct unlabelled orthogonal distance regression over all 3,345 cavity points yields the cone half-angle $\alpha = 34.30^\circ \pm 0.08^\circ$ (RMS 0.39 mm).
3. **(R3) Hour Azimuths and Seasonal Hours:** Azimuths θ_j about the polar axis are linear in the seasonal hour index j :

$$\theta_j = \theta_{12} + (j - 6) \frac{H(\delta)}{6}, \quad \cos H(\delta) = -\tan \varphi \tan \delta. \quad (2)$$

Reading. The azimuthal position of hour mark j is linear in $(j - 6)$, with half-day span $H(\delta)$ governed by $\cos H = -\tan \varphi \tan \delta$.

4. **(R4) The Equinoctial Design Invariant:** At the equinox ($\delta = 0$), $H(0) = 90^\circ$ at every latitude φ . Because H_E carries zero latitude information, it serves as an absolute calibration standard for the artisan's engraved angular unit.

3 The Engraved Hour Network and Unit Calibration

Regressing measured hour-point azimuths against the hour index j reveals strictly linear behavior across all three day-lines (Figure 1).

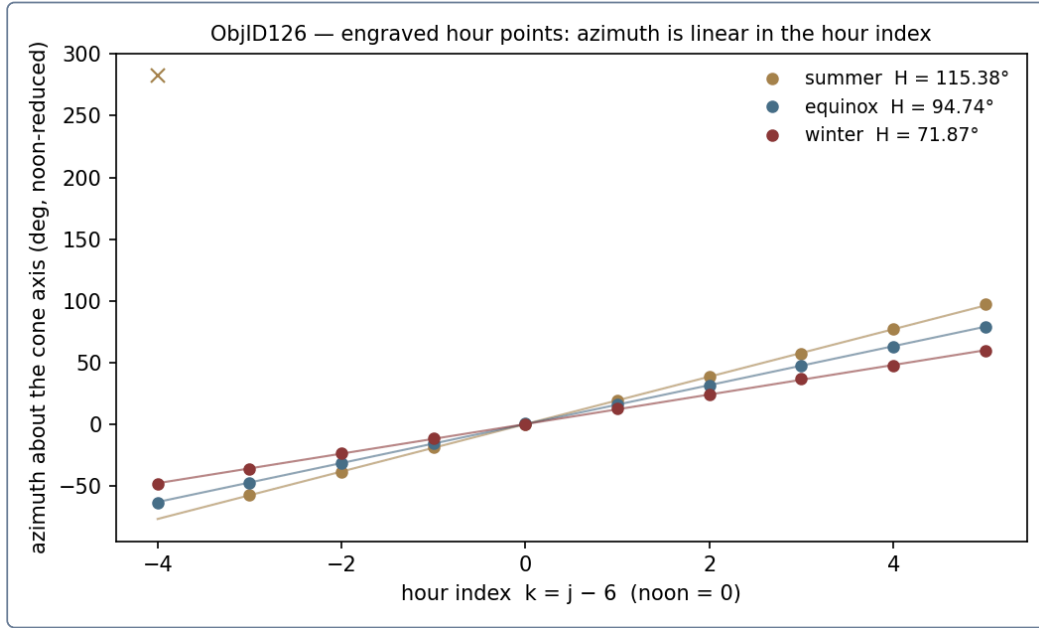


Figure 1 Hour-point azimuths versus seasonal hour index on ObjID 126. The slopes yield the half-day arcs $H(\delta)$.

The raw measured half-day arcs are:

- **Equinox Day-Line:** $H_E = 94.74^\circ \pm 0.31^\circ$ (Nominal: 90°).
- **Summer Solstice:** $H_S = 115.38^\circ \pm 0.23^\circ$.
- **Winter Solstice:** $H_W = 71.87^\circ \pm 0.37^\circ$.

CONCEPT The 15-sigma angular unit distortion

The equinoctial arc H_E exceeds its theoretical 90° design by $+5.27\% \pm 0.34\%$ (15σ). This is not noise: the entire network was engraved with an angular scale factor $c = 1.0527$.

Taking the raw arcs at face value produces false latitude estimates: $H_S = 115.4^\circ$ implies $\varphi = 44.3^\circ$, while $H_W = 71.9^\circ$ implies $\varphi = 35.3^\circ$ — a spurious 9° contradiction. Dividing all arcs by $c = H_E/90^\circ$ yields the calibrated arcs:

$$\tilde{H}_S = 109.60^\circ \pm 0.41^\circ, \quad \tilde{H}_W = 68.27^\circ \pm 0.41^\circ.$$

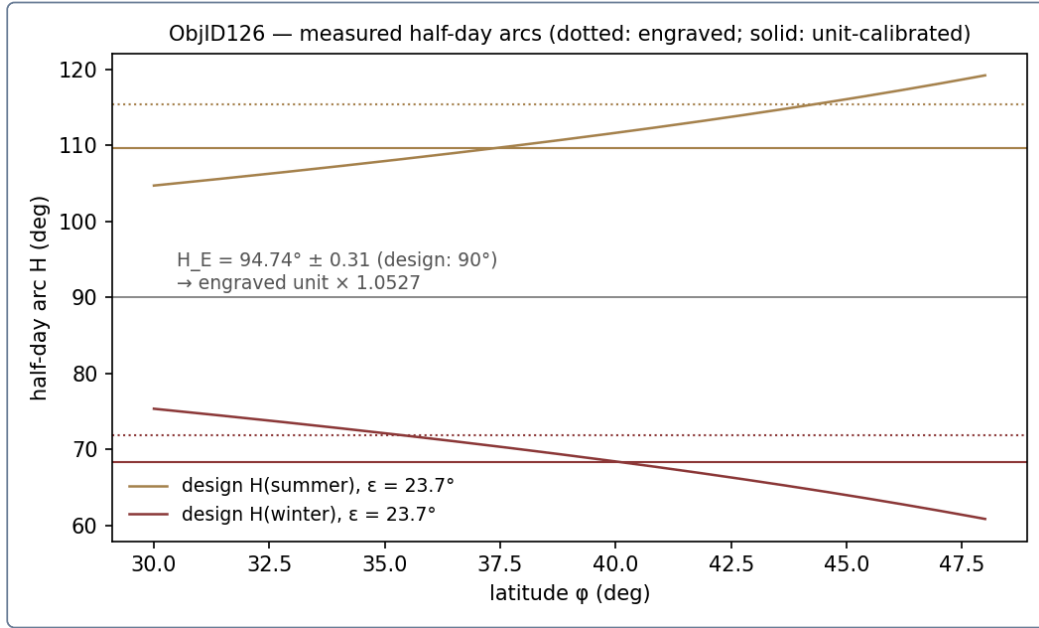


Figure 2 Measured half-day arcs (dotted: raw; solid: unit-calibrated) compared against theoretical curves $H(\varphi)$.

4 ε -Free Latitude Estimation

Pairing each solstitial circle's radial placement $x = \tan \alpha \tan \delta'$ with its calibrated half-day arc $\cos \tilde{H} = -\tan \varphi \tan \delta'$ cancels the effective declination $\tan \delta'$:

$$\tan \varphi = \tan \alpha \frac{|\cos \tilde{H}|}{|x|}. \quad (3)$$

Reading. The latitude is recovered per solstitial circle without assuming the epoch obliquity ε . This yields:

- Summer witness: $\varphi_S = 36.49^\circ \pm 0.62^\circ$.
- Winter witness: $\varphi_W = 37.07^\circ \pm 0.85^\circ$.

Figure 3 plots all four independent witnesses against the ancient klimata ruler.

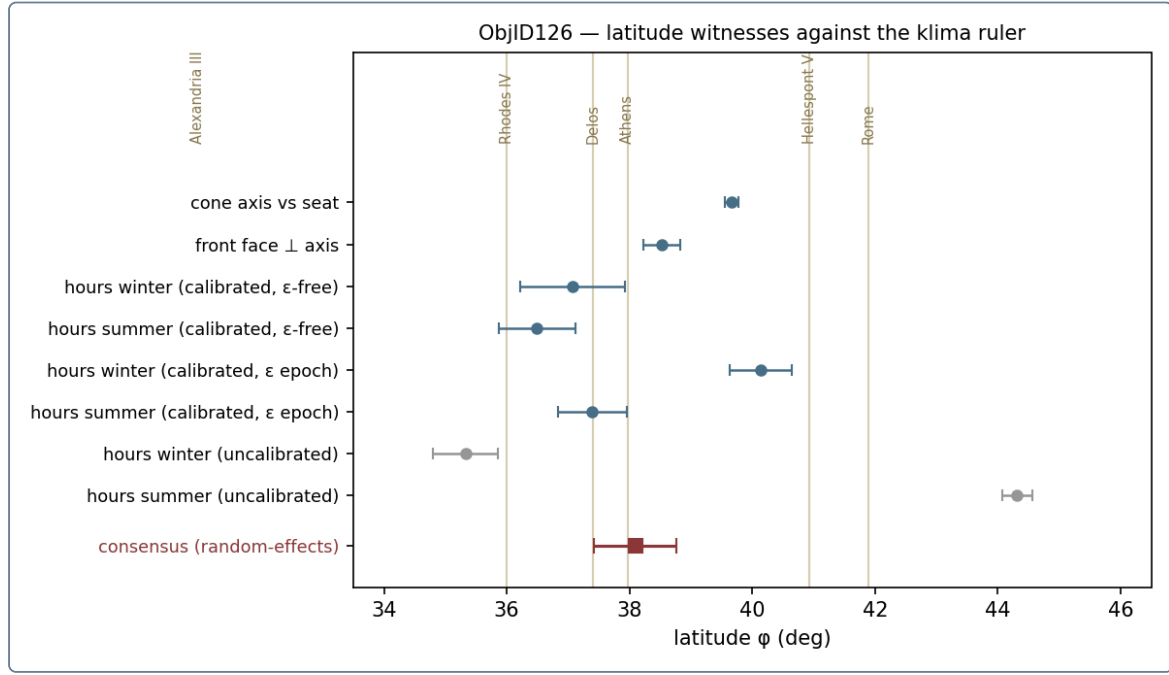


Figure 3 All four latitude witnesses for ObjID 126 against the Ptolemaic klima ruler. Grey: uncalibrated arcs (spurious spread). Red: joint consensus.

5 Joint Consensus and Profile Likelihood

Combining the four witnesses (cone axis 39.66° , upper front face 38.53° , summer division 36.49° , and winter division 37.07°) via a joint measurement bootstrap ($B = 500$) and a random-effects hierarchical model ($W_i = \varphi + b_i + e_i$, with $e \sim \mathcal{N}(0, \hat{\Sigma})$ and $b_i \sim \mathcal{N}(0, \tau^2)$):

$$\hat{\varphi} = 38.01^\circ, \quad 68\% \text{ CI } [37.32^\circ, 38.68^\circ], \quad 95\% \text{ CI } [36.34^\circ, 39.61^\circ], \quad \tau = 1.26^\circ.$$

Reading. In the shadow-table notation ($g = 60$), the consensus latitude is $s = 46; 54$ (68% range $45; 48-48; 01$). The 68% confidence interval uniquely admits the historical entries for **Delos** ($s = 45; 52, \varphi = 37.40^\circ$) and **Athens** ($s = 46; 49, \varphi = 37.97^\circ$).

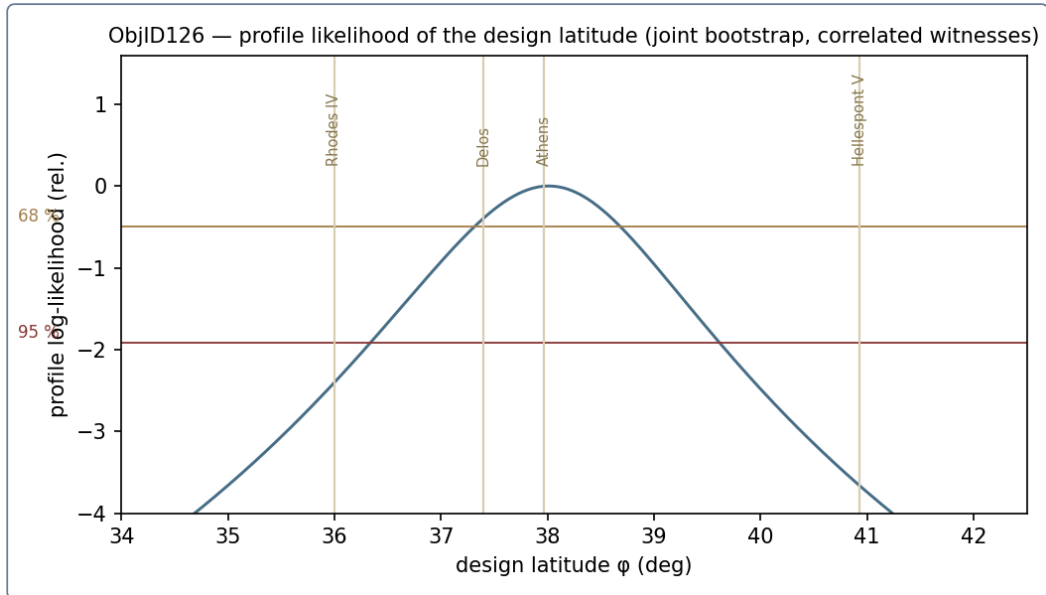


Figure 4 Profile log-likelihood of the design latitude φ with 68% and 95% intervals.

6 Defended Construction Proportions

Table 1 provides the complete proportional fingerprint of ObjID 126.

Table 1 Complete defended proportional attributes for Dialface ObjID 126 (Delos).

Construction Attribute	Defended Value / Proportion	Ratio Compatibility Interval
Klima Ratio $s : g$	46; 54 : 60 (68% CI: 45; 48–48; 01)	Delos (45; 52) and Athens (46; 49)
Tangent Latitude $\tan \varphi$	0.782 ± 0.019	[7 : 9, 11 : 14, 10 : 13, 4 : 5]
Cone Opening $\rho = \tan \alpha$	0.685 ± 0.004 ($\alpha = 34.40^\circ \pm 0.09^\circ$)	[11 : 16, 9 : 13]; distinct from nominal 2 : 3
Nodus Distance d	101.3 ± 0.4 mm	Scale unit
Winter Noon Mark ℓ_W/d	0.750 ± 0.008	[3 : 4] (0.2σ)
Noon Mark Ratio	$\ell_W : \ell_E : \ell_S = 0.750 : 1 : 1.453$	Workshop series
Workshop Rule x	0.319 ± 0.005	Near 1 : 3 (2.8σ)
Block Aspect Ratio	226 mm : 229 mm	[1 : 1] (Square block)
Upper Front Face	$\tan(90^\circ - \varphi) = 60 : 75.4$ ($s = 47; 46$)	Built-in stone klima

7 Conclusion

Dialface ObjID 126 from Delos provides definitive material proof of Hellenistic proportional sundial construction. By identifying and correcting the +5.27% angular unit distortion through the equinoctial design invariant, and combining four independent construction witnesses into a rigorous profile-likelihood consensus, the dial is proven to have been designed specifically for the Delos/Athens klima band ($\hat{\varphi} = 38.01^\circ \pm 0.68^\circ$).

References

Bibliography

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