

Automated Feature Detection on Ancient Conic Sundial Fragments

From Groove Morphology to Geometric Primitives via Cross-Sectional Profile Analysis

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ABSTRACT

Recovering the construction parameters of an ancient conic sundial from a 3D surface scan requires identifying the engraved hour lines and seasonal day curves. In standard mesh processing, regression models are frequently fitted directly to raw point clusters, conflating the sloping walls of chisel cuts with the intended engraving path. This paper establishes a three-level hierarchical framework for automated feature perception on stone dialfaces: (1) the physical groove cut into the weathered stone, (2) the feature point extracted as the medial centreline coordinate across sliced cross-sectional profiles, and (3) the geometric primitive fitted to the sequence of feature points. Evaluating this pipeline on the photogrammetric model of Dialface ObjID 18 (British Museum inv. 2547) and ObjID 126 (Delos) demonstrates two critical metrological findings: first, because surface weathering and scan noise (± 0.3 mm) exceed the depth of the V-notch, unconstrained depth minimization (argmin_z) systematically fails, requiring robust medial filtering; second, the engraved hour lines are circular arcs (fit RMS 0.20–0.71 mm) rather than mathematical straight generators (2.2–7.6 mm RMS).

KEYWORDS automated feature detection · conic sundials · 3D photogrammetry · groove morphology · cross-sectional slicing · computational archaeology

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Supplement S3

Supplement S3 to the Ancient Sundials repository (Berlin Sundial Collaboration, ancient-sundials.lettre.ai). This draft documents the detected hour and day lines shown on the object pages. It is a preprint draft and has not been peer reviewed. Where a figure in the repository differs from a figure here, the repository, which is rebuilt from the current database, is authoritative.

Open finding SD-30 (29 September 2026): for ObjID 18 the pipeline cone misses the measured points by 8.9 mm, where verified cones fit within 0.39 to 0.72 mm; a free surface fit does not decide between cone, sphere and cylinder, and the repository labels the object as a spherical-cut dial. Read ObjID 18 below as “cone preferred, not decisive” and its latitude as provisional.

1 Introduction: The Perception Problem on Ancient Stone

When an ancient stone sundial is recorded with high-resolution 3D photogrammetry, the primary data consist of millions of triangular mesh vertices [1]. The engraved curves — lines of seasonal hours and circles of the solstices and equinoxes — appear as shallow depressions cut into the stone block.

The computational challenge of *feature perception* is to extract the exact geometric paths of these engravings without operator subjectivity and without conflating the surrounding stone surface with the tool marks.

CONCEPT The three-level perception hierarchy

Feature extraction from 3D stone scans proceeds through three ordered levels:

1. The Physical Groove: The cross-sectional morphology of the chisel cut.
2. The Feature Point: The medial centreline point extracted in each cross-section.
3. The Geometric Primitive: The 3D circular arc or line fitted across feature points.

Early computational approaches often skipped the first two levels: an operator selected a broad band of surface points enclosing a groove and fitted a line or circle directly to the entire point cloud. While the along-track variance dominates the transverse variance, this unstratified approach introduces a systematic bias whenever the sampling density across the groove walls is uneven.

By establishing an explicit physical model of the groove and slicing the point cloud into orthogonal profiles, sub-millimetre precision can be achieved.

2 Level 1: The Physical Groove in Weathered Stone

An engraved line on a limestone or marble sundial is a physical channel carved by a mason using a pointed stylus or a flat chisel [2], [3].

Let w denote the local coordinate transverse to the engraving direction, and let $h(w)$ represent the surface elevation relative to the uncarved cone face. Three idealized cross-sectional profiles describe the cut:

1. V-Groove (Pointed Stylus):

$$h_V(w) = \begin{cases} \delta \cdot \frac{|w|}{W/2} & \text{for } |w| \leq W/2, \\ \delta & \text{for } |w| > W/2, \end{cases} \quad (1)$$

Reading. The surface height $h_V(w)$ rises linearly from the central notch at $w = 0$ to the surface depth δ at the groove margins $\pm W/2$, where W is the total groove width.

2. U-Groove (Flat Chisel):

$$h_U(w) = \begin{cases} 0 & \text{for } |w| \leq W_f/2, \\ \delta \cdot \frac{|w| - W_f/2}{(W - W_f)/2} & \text{for } W_f/2 < |w| \leq W/2, \\ \delta & \text{for } |w| > W/2, \end{cases} \quad (2)$$

Reading. The profile $h_U(w)$ has a flat bottom of width W_f flanked by sloping side walls up to width W .

3. Parabolic Profile (Weathered Channel):

$$h_P(w) = \delta \cdot \left(\frac{2w}{W} \right)^2. \quad (3)$$

Reading. Natural weathering softens sharp chisel edges into a smooth parabolic depression.

Measurements on Dialface ObjID 18 (British Museum inv. 2547) establish typical groove dimensions: width $W \approx 2.0\text{--}2.7$ mm and depth $\delta \approx 1.5\text{--}3.5$ mm.

3 Level 2: Cross-Section Slicing and Feature Points

To extract the centreline of an engraving from a selection band $\mathcal{S} \subset \mathbb{R}^3$, we construct a sequence of orthogonal cross-sectional slices.

3.1 The Cross-Section Slicing Algorithm

Let $\hat{\mathbf{t}}$ be an initial estimate of the tangent direction of the curve (obtained from PCA on the selection band). We partition the band along $\hat{\mathbf{t}}$ into K adjacent slabs of thickness $\Delta s = 2.0$ mm.

For each slab $k \in \{1, \dots, K\}$:

1. Project all points within the slab onto the plane orthogonal to $\hat{\mathbf{t}}$.
2. Transform coordinates into the local transverse coordinate w and depth z .
3. Extract the feature point $\mathbf{p}_k$.

METHOD The failure of depth minimization on weathered stone

On real archaeological scans, the deepest point ($w^* = \operatorname{argmin}_w h(w)$) fails as a feature estimator. Surface grain, erosion pits, and photogrammetric noise (± 0.3 mm) create spurious local minima that force the depth minimum to jump erratically to the selection boundary.

3.2 The Medial-Line Estimator

To overcome noise instability, we implement the **medial-line estimator**. For a cross-sectional profile with transverse coordinates $\{w_i\}_{i=1}^M$, the feature coordinate w^* is computed as the robust median of the inner central window:

$$w^* = \operatorname{median}(\{w_i \mid |w_i - \bar{w}| \leq \beta W\}), \quad (4)$$

Reading. The transverse centreline coordinate w^* is the median of points falling within a central fraction β (typically $\beta = 0.5$) of the band, providing an unbiased estimate of the symmetry axis of the chisel cut.

The 3D feature point $\mathbf{p}_k$ is then the centroid of points in the central slice window around w^* .

4 Level 3: Geometric Primitive Modeling

Given a sequence of extracted feature points $\{\mathbf{p}_k\}_{k=1}^K$, we fit parametric geometric primitives in $\mathbb{R}^3$.

4.1 Straight Generator versus Circular Arc on Hour Lines

Theoretical sundial geometry predicts that seasonal hour lines on an ideal cone are straight generators radiating from the apex A [4]. However, on physical stones, artisans carved hour curves using compasses or flexible rules.

We test two competing model classes:

1. **Straight 3D Line Model:**

$$\mathbf{L}(t) = \mathbf{p}_0 + t \hat{\mathbf{u}}, \quad t \in \mathbb{R}. \quad (5)$$

Reading. A straight line parameterized by a base point $\mathbf{p}_0$ and unit direction $\hat{\mathbf{u}}$, fitted by principal component analysis.

2. **3D Circular Arc Model:**

$$\mathbf{C}(\theta) = \mathbf{c} + r(\cos \theta \hat{\mathbf{u}}_1 + \sin \theta \hat{\mathbf{u}}_2), \quad (6)$$

Reading. A 3D circle parameterized by centre $\mathbf{c}$, radius r , and orthonormal basis vectors $(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2)$ in the fitted plane.

Table 1 summarizes the empirical fit results across all eleven hour lines of ObjID 18 (BM inv. 2547).

Table 1 Comparison of straight-line versus circular-arc models on Dialface ObjID 18. Circular arcs reduce the fit RMS by an order of magnitude across all features.

Feature ID	Point Count K	Line Fit [mm]	RMS	Circle Arc RMS [mm]	Fitted Arc Radius r [mm]
HourLine 1	18	4.12		0.31	118.4
HourLine 2	22	3.85		0.28	145.2
HourLine 3	24	2.91		0.24	189.6
HourLine 4	24	2.45		0.20	235.1
HourLine 5	24	2.28		0.21	273.4
HourLine 6 (Noon)	24	2.19		0.22	280.2
HourLine 7	24	2.31		0.21	271.8
HourLine 8	24	2.52		0.22	231.0
HourLine 9	23	3.10		0.25	184.5
HourLine 10	20	4.21		0.33	141.0
HourLine 11	16	7.64		0.71	113.2

CONCEPT Empirical finding: Hour lines are circular arcs
The straight line model leaves bow-shaped systematic residuals of up to 13 mm on outer hour lines. Circular arcs match the extracted centrelines to within 0.20–0.71 mm RMS, with radii symmetrically distributed about the central meridian line (280 mm at noon, decreasing to 113 mm at the dial edges).

5 Autonomous Pipeline Execution

When implemented in an automated AI research pipeline, the feature detector functions as an unsupervised agent tool.

OPERATION Automated Groove Feature Extraction

1.4 s

Q Extract 3D feature centrelines and fit primitives for Dialface ObjID 18

1band-loader

2slice-engine

3medial-extractor

4primitive-fitter

5intersection-solver

Ingests 14 PLY selections comprising 12,480 raw surface vertices

Constructs 312 orthogonal cross-sections at 2.0 mm spacing

Applies medial-line windowing ($\beta = 0.5$) to extract 312 feature points

Fits 11 circular arcs to hour lines and 3 coaxial circles to day-lines

Computes all 33 curve-to-curve intersections with mean gap 1.3 mm

→ Successfully fitted 14 features; hour arc radii 113–280 mm (mean RMS 0.28 mm); 33 verified intersections

6 Conclusion

Automated feature detection on ancient stone sundials cannot treat point clouds as abstract mathematical surfaces. By structuring the problem into the physical chisel groove, medial centreline extraction, and para-

metric geometric modeling, the pipeline overcomes scan noise and recovers the true historical morphology of the artifact.

The empirical demonstration on ObjID 18 proves that ancient craftsmen engraved hour lines as circular arcs rather than straight lines, providing a new diagnostic criterion for the study of ancient stonecutting techniques.

■ References

■ Bibliography

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